IL NUOVO CIMENTO Vol. ?, N. ? ?

Light deflection and time delay in the gravitational field of a spinning body

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Summary. — In this paper we consider the possibility of measuring the corrections induced by the square of the parameter a_g of the Kerr metric to the general relativistic deflection of electromagnetic waves and time delay in an Earth based experiment. It turns out that at laboratory scale the rotational effects exceed definitely the gravitoelectric ones which are totally negligible. By using a small rapidly rotating sphere as gravitating source on the Earth the deflection of a grazing light ray amounts to 10^{-13} rad and the time delay is proportional to 10^{-23} s. These figures are determined by the upper limit in the attainable values of a_g due to the need of preventing the body from exploding under the action of the centrifugal forces. Possible criticisms to the use of the Kerr metric at a_g^2 level are discussed.

PACS 95.30 – Relativity and gravitation.
PACS 04.20 – Fundamental problems and general formalism.

1. – Introduction

The effect of the proper angular momentum J of a body of mass M, assumed as source of the gravitational field, is accounted for, in General Relativity, by the parameter $a_g = \frac{J}{Mc}$, where c is the speed of light in vacuum, entering, e.g., the Kerr metric. It has the dimensions of a length and is of order $\mathcal{O}(c^{-1})$. The other characteristic length is the Schwarzschild radius $R_s = \frac{2GM}{c^2}$, where G is the Newtonian gravitational constant, which enters both Schwarzschild and Kerr metrics.

Traditionally, in almost all the applications of the Kerr metric to the motion of test particles or electromagnetic waves in the space–time of a central spinning mass the square of a_g, which is of order $\mathcal{O}(c^{-2})$ and enters the diagonal components of the metric tensor, is neglected. However, in a recent stimulating letter in ref. [1] it is suggested that, instead, it would be better to account for it in view of possible experimental setups on the Earth. Indeed, at laboratory scale, while the effects due to R_s, of order $\mathcal{O}(c^{-2})$, are certainly negligible because of the smallness of the product GM, those due to a_g^2, of order $\mathcal{O}(c^{-2})$ as well, could become interesting because, for certain symmetric geometries of the source body, it depends only on the fourth power of its radius R and the square of its angular velocity Ω.

In this paper we want to explore this scenario by working out the contributions of a_g^2 to two classical tests of General Relativity: the deflection of the electromagnetic waves and the time delay [2]. We will consider the motion of photons in the equatorial plane of a central rotating body at rest. In a number of papers these general relativistic effects have been worked out for various systems of gravitating bodies endowed with mass–monopole, spin–dipole and time–dependent mass and spin multipoles for different states of motions both of the source of electromagnetic waves and the observer [3, 4, 5, 6, 7, 8] by means of a variety of general mathematical approaches, but the influence of a_g^2 has never been considered.

A possible objection to the present calculations could be that they are based on the use of the Kerr metric in representing the gravitational field of an extended object at the a_g^2 order. Indeed, it should be reminded that, up to now, nobody has been successful in extending the Kerr metric from the empty space to the interior of a body. In fact, the field of a real rotating body would be endowed of various multipolar mass moments, etc. and if we do not know what is the matter source, as in the case of the Kerr metric, we do not know how to cope with them([1]). Then, one could wonder if, at the level of approximation considered, the Kerr metric and the derived consequences have physical meaning. However, notice that in ref. [9] the author use the full Kerr metric in deriving the general relativistic corrections of the Sagnac effect. In ref. [10] the Kerr metric is used in order to sketch a possible Earth based laboratory experiment aimed to testing gravitomagnetism at order a_g. Moreover, recently some efforts are or will be directed (M. McCallum, private communication, 2002) towards possible extensions of the Kerr metric to various interior matter sources. Many efforts have been dedicated to the construction of sources which could represent some plausible models of, e.g., stars. In ref. [11] the authors have tried to connect the external Kerr metric to the multipolar structure of various types of stars assumed to be rigidly and slowly rotating. Terms of greater than the second order in the angular velocity were neglected. In ref. [12] a physically reasonable fluid source for the Kerr metric has been obtained. For previous attempts to construct sources for the Kerr metric see refs. [13, 14]. In ref.[15] a general class of solutions of Einstein's equations for a slowly rotating fluid source, with supporting internal pressure, is matched to the Kerr metric up to and including first order terms in angular speed parameter. So, in this context, our calculations could be useful in order to give an idea of what could happen in this case and, more generally, if experiments testing different scenarios at order $\mathcal{O}(J^2)$ are realistically conceivable.

A possible subject for further analysis could be an investigation of the correspondence with the Post–Newtonian expansion at the J^2 level. At the first order in J the Post–Newtonian expansion accounts for the angular momentum of the source in the off–diagonal components g_{0i} $i = 1, 2, 3$ via a c^{-3} term([2]). So, at a first glance, it could be guessed that the Post–Newtonian expansion should account for J^2 with a c^{-6} term which, instead, is absent in the Kerr metric. Indeed, as we will see, our effects depends only on a_g^2.

Recently, in ref. [16] an approximated solution of the Einstein field equations for

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([1]) However, in the present case we could argue that, whatever the description of the non–sphericity of the central mass could be, it should not affect in any relevant way the obtained results, as confirmed a posteriori by the presented calculations.
([2]) This may explain the fact that Kerr metric is commonly accepted in describing the Lense–Thirring effect and the gravitomagnetic clock effects which are linear in J.

a rotating, weakly gravitating body has been found. It seems promising because both external and internal metric tensors have been consistently found, together an appropriate source tensor. Moreover, the mass and the angular momentum per unit mass are assumed to be such that the mass effects are negligible with respect to the rotation effects. As a consequence, only quadratic terms in the body's angular velocity has been retained. One main concern is that it is not clear if there is a real mass distribution able to generate the found source tensor.

As can be seen, the subject seems to be rather open and, in the author's opinion, worth of investigation, at least in order to get some estimates of the orders of magnitude involved.

The paper is organized as follows: in section 2 we will derive the geodesic motion of a test particle of mass m in the equatorial plane of a rotating body. In section 3 we will specialize it to the photons and will work out the deflection of electromagnetic waves due to a_g^2. In section 4 its effect on the time delay is examined. Section 5 is devoted to the discussion of the obtained results and to possible applications to Earth laboratory experimental scenarios.

2. – Plane geodesics in the Kerr metric

Let us consider, for the sake of concreteness, a spherically symmetric rigid body of mass M, radius R and proper angular momentum J directed along the z axis of an asymptotically inertial frame $K\{x, y, z\}$ whose origin is located at the center of mass of the body. By adopting, as usual, the coordinates

$$x^0 = ct, \tag{1}$$
$$x^1 = r, \tag{2}$$
$$x^2 = \theta, \tag{3}$$
$$x^3 = \phi, \tag{4}$$

the components of the Kerr metric tensor are [17]

$$g_{00} = 1 - \frac{R_s r}{\varrho^2}, \tag{5}$$
$$g_{11} = -\frac{\varrho^2}{\Delta}, \tag{6}$$
$$g_{22} = -\varrho^2, \tag{7}$$
$$g_{33} = -\sin^2\theta \left[r^2 + a_g^2 + \frac{R_s r}{\varrho^2} a_g^2 \sin^2\theta\right], \tag{8}$$
$$g_{03} = \frac{R_s r}{\varrho^2} a_g \sin^2\theta, \tag{9}$$

with

$$\varrho^2 = r^2 + a_g^2 \cos^2\theta, \tag{10}$$
$$\Delta = r^2 - R_s r + a_g^2. \tag{11}$$

For a spherical body spinning at angular velocity Ω and with moment of inertia $I = \frac{2}{5}MR^2$ the characteristic length a_g becomes

$$a_g = \frac{I\Omega}{Mc} = \frac{2}{5}\frac{R^2\Omega}{c}. \tag{12}$$

It is important to note that neither the Newtonian gravitational constant G nor the mass M are present in a_g which is determined only by the geometrical and kinematical properties of the body. This is a feature which will turn out to be very relevant in proposing experiments at laboratory scale.

By putting

$$\varepsilon = \frac{R_s}{r} \tag{13}$$

$$\alpha = \frac{a_g}{r}, \tag{14}$$

the line element for a test particle of mass m moving in the space–time of the central body in its equatorial plane, i.e. at fixed $\theta = \frac{\pi}{2}$, is given by [18]

$$c^2 = (1-\varepsilon)(\dot{x}^0)^2 - \frac{(\dot{r})^2}{1-\varepsilon+\alpha^2} - r^2(1+\alpha^2+\varepsilon\alpha^2)(\dot{\phi})^2 + 2\varepsilon\alpha r\dot{x}^0\dot{\phi}. \tag{15}$$

In eq.(15) the overdot denotes the derivative with respect to proper time τ.

From the Lagrangian

$$\mathcal{L} = \frac{m}{2}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = \frac{m}{2}\left[(1-\varepsilon)(\dot{x}^0)^2 - \frac{(\dot{r})^2}{1-\varepsilon+\alpha^2} - r^2(1+\alpha^2+\varepsilon\alpha^2)(\dot{\phi})^2 + 2\varepsilon\alpha r\dot{x}^0\dot{\phi}\right], \tag{16}$$

since the field is stationary and axially symmetric, it is possible to obtain the following constants of motion

$$\frac{\partial\mathcal{L}}{\partial\dot{x}^0} = m(1-\varepsilon)\dot{x}^0 + m\varepsilon\alpha r\dot{\phi} = K \tag{17}$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\phi}} = -mr^2(1+\alpha^2+\varepsilon\alpha^2)\dot{\phi} + m\varepsilon\alpha r\dot{x}^0 = H. \tag{18}$$

Let us write eq.(15) as

$$\left(\frac{c}{\dot{\phi}}\right)^2 = (1-\varepsilon)\left(\frac{\dot{x}^0}{\dot{\phi}}\right)^2 - \frac{1}{1-\varepsilon+\alpha^2}\left(\frac{\dot{r}}{\dot{\phi}}\right)^2 - r^2(1+\alpha^2+\varepsilon\alpha^2) + 2\varepsilon\alpha r\left(\frac{\dot{x}^0}{\dot{\phi}}\right). \tag{19}$$

In view of an application to possible experimental scenarios at laboratory scale, it is very instructive to note that

$$\alpha^2 \sim \frac{1}{c^2} \tag{20}$$

$$\varepsilon \sim \frac{GM}{c^2} \tag{21}$$

$$\varepsilon^2 \sim \frac{G^2M^2}{c^4} \tag{22}$$

$$\varepsilon\alpha \sim \frac{GM}{c^3} \tag{23}$$

$$\varepsilon\alpha^2 \sim \frac{GM}{c^4}. \tag{24}$$

This implies that, at laboratory scale, the terms with the square of the characteristic length a_g may give not negligible contributions with respect to the other terms which are quite negligible because of the presence of GM and/or because they are of order $\mathcal{O}(c^{-n})$, $n \geq 3$.

From eqs.(17)-(18), by keeping, in a first step, only terms of order $\mathcal{O}(c^{-2})$, it is possible to obtain

$$\left(\frac{\dot{x}^0}{\dot{\phi}}\right) = \frac{K}{H}\frac{r^2(\alpha^2+1)}{(\varepsilon-1)} \tag{25}$$

$$\frac{1}{\dot{\phi}} = \frac{m}{H}\frac{r^2(1+\alpha^2-\varepsilon)}{(\varepsilon-1)}. \tag{26}$$

With

$$u \equiv \frac{1}{r} \tag{27}$$

$$\left(\frac{\dot{r}}{\dot{\phi}}\right) = \frac{dr}{d\phi} = -\frac{1}{u^2}\frac{du}{d\phi} \tag{28}$$

and by inserting eqs.(25)-(26) in eq.(19) one obtains, to order $\mathcal{O}(c^{-2})$

$$\left(\frac{du}{d\phi}\right)^2 = \left(\frac{K^2-m^2c^2}{H^2}\right) - u^2 + \frac{m^2c^2}{H^2}R_s u - 3\frac{m^2c^2}{H^2}a_g^2u^2 + 3\left(\frac{K}{H}\right)^2 a_g^2u^2 + R_s u^3 - 2a_g^2u^4. \tag{29}$$

By taking the derivative of eq.(29) it can be obtained

$$\frac{d^2u}{d\phi^2} = -u + \frac{m^2c^2R_s}{2H^2} - 3\frac{m^2c^2}{H^2}a_g^2u + 3\left(\frac{K}{H}\right)a_g^2u + \frac{3}{2}R_s u^2 - 4a_g^2u^3. \tag{30}$$

Note that in eq.(30) the first three terms of the right hand side are of order $\mathcal{O}(c^0)$ and the following three terms are of order $\mathcal{O}(c^{-2})$.

3. – The deflection of light

In order to cope with the case of photons having zero rest mass, let us pose $m = 0$ in eqs.(29)-(30) so to obtain

$$\left(\frac{du}{d\phi}\right)^2 = \left(\frac{K}{H}\right)^2 - u^2 + 3\left(\frac{K}{H}\right)^2 a_g^2u^2 + R_s u^3 - 2a_g^2u^4, \tag{31}$$

$$\frac{d^2u}{d\phi^2} = -u + 3\left(\frac{K}{H}\right)^2 a_g^2u + \frac{3}{2}R_s u^2 - 4a_g^2u^3. \tag{32}$$

In the following, without loss of generality, we will assume to count the angle ϕ from the point of closest approach so that $u(\phi=0)=u_{max}=\frac{1}{r_{min}}\equiv\frac{1}{b}$.

3·1. *The effect of* $\mathbf{a_g^2}$. – In order to calculate the effect of a_g^2, which is of order $\mathcal{O}(c^{-2})$, let us drop the terms in R_s in eqs.(31)-(32): they become([3])

$$\left(\frac{du}{d\phi}\right)^2=\left(\frac{K}{H}\right)^2-u^2+3\left(\frac{K}{H}\right)^2a_g^2u^2-2a_g^2u^4, \tag{33}$$

$$\frac{d^2u}{d\phi^2}=-u+3\left(\frac{K}{H}\right)^2a_g^2u-4a_g^2u^3. \tag{34}$$

By evaluating eq.(33) for $\phi=0$ it is possible to obtain K/H. Indeed, at the closest approach we have $u=u_{max}$ and $\frac{du}{d\phi}=0$, so that, from eq.(33) at order $\mathcal{O}(c^0)$

$$\left(\frac{K}{H}\right)^2\sim u_{max}^2. \tag{35}$$

Then, eq.(34) becomes

$$\frac{d^2u}{d\phi^2}+(1-3u_{max}^2a_g^2)u+4a_g^2u^3=0. \tag{36}$$

In order to solve this equation, it is useful to pose

$$\omega_0^2\equiv(1-3u_{max}^2a_g^2), \tag{37}$$

$$\gamma\equiv\frac{4a_g^2}{(1-3u_{max}^2a_g^2)}\sim 4a_g^2. \tag{38}$$

This allows to write eq.(36) as

$$\frac{d^2u}{d\phi^2}+\omega_0^2u+\gamma\omega_0^2u^3=0. \tag{39}$$

The solution of an equation of the form of eq.(39) is given by [19]

$$u=u_{max}\left(1-\gamma\frac{u_{max}^2}{32}\right)\cos\omega\phi+\frac{\gamma u_{max}^3}{32}\cos 3\omega\phi, \tag{40}$$

with

$$\omega^2=\frac{\omega_0^2}{1-\frac{3}{4}\gamma u_{max}^2}. \tag{41}$$

At order $\mathcal{O}(c^{-2})$ it reads

$$\omega^2\sim 1; \tag{42}$$

([3]) This approximation is fully satisfied for laboratory scale bodies, as it will become clear later.

then eq.(40) becomes

$$u = u_{max}\left(1 - \gamma\frac{u_{max}^2}{32}\right)\cos\phi + \frac{\gamma u_{max}^3}{32}\cos 3\phi. \tag{43}$$

From eq.(43) it can be easily obtained the deflection angle. Indeed, let us assume that the photon flies away at infinity, i.e. $u \to 0$, not for $\overline{\phi} = \frac{\pi}{2}$, as it should be the case for the straight line propagation in flat Minkowskian space–time, but for $\overline{\phi}^{\prime} = \frac{\pi}{2} + \delta_{a_g^2}$. So, eq.(43) yields

$$\delta_{a_g^2} = -\frac{\gamma u_{max}^2}{32}\left(1 + \frac{\gamma u_{max}^2}{32}\right) \sim -\frac{\gamma u_{max}^2}{32} \sim -\frac{a_g^2}{8b^2} = -\frac{J^2}{8M^2c^2b^2} = -\frac{1}{50}\frac{R^4\Omega^2}{c^2b^2}. \tag{44}$$

4. – The time delay

The coordinate time interval between the emission and the reception of an electromagnetic signal at two different points A and B (one-way travel) can be written as [20]

$$dt = \frac{\sqrt{(g_{0i}g_{0j} - g_{ij}g_{00})dx^i dx^j}}{cg_{00}}. \tag{45}$$

In it the Latin indices run from 1 to 3 and the Einstein summation convention is adopted. For the Kerr metric at $\theta = \frac{\pi}{2}$ eq.(45) becomes

$$dt = \frac{\sqrt{({g_{03}}^2 - g_{33}g_{00})d\phi^2 - g_{11}g_{00}dr^2}}{cg_{00}} \sim \frac{\sqrt{-g_{33}d\phi^2 - g_{11}dr^2}}{c\sqrt{g_{00}}} \sim \frac{\sqrt{r^2(1+\alpha^2)d\phi^2 + \frac{dr^2}{1-\varepsilon+\alpha^2}}}{c\sqrt{1-\varepsilon}}. \tag{46}$$

Eq.(46) has been obtained by neglecting g_{03}, proportional to $\varepsilon\alpha$, and $\varepsilon\alpha^2$ in g_{33}. By assuming the polar axis as x axis parallel to the (almost) straight line motion of the photons, we can neglect $r^2 d\phi^2$ with respect to dr^2 and we can pose $dr \sim dx$, $r \sim \sqrt{x^2+b^2}$. With these approximation and by neglecting, as usual, the terms proportional to ε^2 and $\varepsilon\alpha^2$, eq.(46) yields

$$dt = \left[1 + \frac{R_s}{\sqrt{x^2+b^2}} - \frac{a_g^2}{2(x^2+b^2)}\right]\frac{dx}{c}. \tag{47}$$

By integrating eq.(47) from x_A to x_B it can be obtained

$$\Delta t = \Delta t_0 + \Delta t_{\mathrm{GE}} + \Delta t_{a_g^2} \tag{48}$$

with

$$\Delta t_0 = \frac{x_B - x_A}{c}, \tag{49}$$

$$\Delta t_{\mathrm{GE}} = \frac{R_s}{c}\ln\left|\frac{x_B + \sqrt{{x_B}^2+b^2}}{x_A + \sqrt{{x_A}^2+b^2}}\right|, \tag{50}$$

$$\Delta t_{a_g^2} = -\frac{a_g^2}{2cb}\left[\arctan\left(\frac{x_B}{b}\right) - \arctan\left(\frac{x_A}{b}\right)\right]. \tag{51}$$

Eq.(49) is the ordinary time interval in the flat Minkowski space–time. Eq.(50) is the well known gravitoelectric Shapiro time delay [21] due to the spherical mass M supposed non–rotating. Eq.(51) represents the new term due to a_g^2 of the Kerr metric.

5. – Discussion

Here we investigate the two relativistic effects worked out and examine the possibility of measure them in terrestrial experiments([4]).

5·1. *The deflection of light*. – Eq.(44) shows many interesting features

- It is of order $\mathcal{O}(c^{-2})$, as the well known gravitoelectric deflection angle [2]

$$\delta_{\rm GE} = 2\frac{R_s}{b} = 4\frac{GM}{c^2 b} \tag{52}$$

- It is independent of the frequency of the electromagnetic radiation, as $\delta_{\rm GE}$
- It is opposite in sign with respect to $\delta_{\rm GE}$ acting as a diverging effect
- It does not contain neither the Newtonian gravitational constant G nor the mass M of the central body, a feature that may be of great help in an Earth laboratory experiment
- It depends on b^{-2}, contrary to $\delta_{\rm GE}$ which, instead, depends on b^{-1}.
- It is insensitive to the sense of rotation of the central body: it is a pity since such a feature could have been useful in order to generate some particular signature.

Let us investigate what could happen in a possible Earth laboratory experiment. E.g., if we assume as central body a small sphere of 2.5 cm radius, mass of 111 g and spinning at([5]) 4×10^4 rad s^{-1} we would have

$$R_s^{\rm sphere} = 1.647 \times 10^{-28} \text{ m}, \tag{53}$$

$$a_g^{\rm sphere} = 3.61 \times 10^{-8} \text{ m}. \tag{54}$$

For a grazing light ray, which means that

$$\delta_{a_g^2} = -\frac{1}{50}\left(\frac{\Omega R}{c}\right)^2 = -\frac{1}{50}\left(\frac{v_{\rm per}}{c}\right)^2, \tag{55}$$

the deflection would amount to

$$\delta_{\rm GE}^{\rm sphere} = 1.31 \times 10^{-26} \text{ rad} = 2.717 \times 10^{-21} \text{ asec}, \tag{56}$$

$$\delta_{a_g^2}^{\rm sphere} = -2.606 \times 10^{-13} \text{ rad} = -5.37 \times 10^{-2}\ \mu\text{asec}. \tag{57}$$

([4]) It is important to notice that in an astrometric scenario, e.g., at the limb of the Sun or of a compact star, the gravitoelectric term of order $\mathcal{O}(c^{-2})$ and the gravitomagnetic term of order $\mathcal{O}(c^{-3})$ are far larger than the terms due to a_g^2 and the approximations used until now are no longer valid. See, e.g., the refs. [22, 23, 24, 25].

([5]) Indeed, according to ref. [26], the maximum peripheral linear speed attainable by a spinning body is $v_{max} \sim 10^3$ m s^{-1}.

In this case the deflection due to a_g^2 is 13 orders of magnitude larger than the gravitoelectric deflection.

5·2. *The time delay.* – Eq.(51) exhibits the following characteristics

- It is of order $\mathcal{O}(c^{-3})$, as Δt_{GE}
- It is independent of the frequency of the electromagnetic radiation, as Δt_{GE}
- It does not contain neither the Newtonian gravitational constant G nor the mass M of the central body, contrary to Δt_{GE}
- Its amplitude depends on b^{-1}, contrary to Δt_{GE} which depends only on the characteristic length R_s.
- It is insensitive to the sense of rotation of the central body

At laboratory scale, for the same sphere as before, we have

$$\Delta t_{\mathrm{GE}}^{\mathrm{sphere}} \propto 5.49 \times 10^{-37} \text{ s}, \tag{58}$$

$$\Delta t_{a_g^2}^{\mathrm{sphere}} \propto -8.7 \times 10^{-23} \text{ s}. \tag{59}$$

Also for the time delay the correction due to a_g^2 is 14 orders of magnitude larger than the gravitoelectric one. However, the detection of a time difference of the order of 10^{-23} s is presently out of discussion: suffice it to say that 10^{-23} s is the typical lifetime of a strongly decaying particle (e.g. ρ or N^*).

5·3. *Conclusion.* – In this paper we have calculated the effect of the square of the parameter a_g entering the Kerr metric on the general relativistic deflection of electromagnetic waves and the time delay in the gravitational field of a central rotating source at rest.

Then, we have analysed those two effects in a possible experimental scenario in an Earth based laboratory with a small spinning sphere. Due to their extreme smallness, we have neglected the terms involving GM. The maximum attainable linear velocity $v_{\max} \sim 10^3$ m s^{-1}, due to the effects of the centrifugal forces, puts severe upper constraints to the magnitude of a_g in a laboratory experiment. The derived effects are to small to be detected.

The calculations performed here loose their validity in an astrophysical context because, in this case, neglecting the other terms proportional to GM with respect to a_g^2 is no more allowed.

Acknowledgements I wish to thank S.M. Kopeikin, A. Tartaglia, M.L. Ruggiero and M. MacCallum for useful remarks and insightful discussions. I am grateful also to B. Mashhoon for the references kindly given me. Thanks also to L. Guerriero for his supporting me in Bari.

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